Maths Assessment Levels

Counting and Cardinality

	M4	M5	M6	M7	M8	M9
COUNTING	Counting: Chanter - *Learning Trajectories*	Counting: Reciter - *Learning Trajectories*	Counting: Reciter - *Learning Trajectories*	Counting: Reciter (10) - *Learning Trajectories*	Counting: Corresponder Counting: Counter (Small Numbers) - *Learning Trajectories*	Counting: Producer (Small Numbers) - *Learning Trajectories*
	Chants number words in "sing-song" fashion and may run them together. The number words may be indistinguishable from one another ('onetwothree'). May begin a nonverbal object "counting" such as copying an adult's item-by-item placement of objects.	Verbally counts with distinct words, not necessarily in the correct order to "three" (if non-verbal, uses visual supports to communicate). If knows more number words than number of objects, rattles them off quickly at the end; if more objects, "recycles" number words (inflexible list exhaustion)	Verbally counts with distinct words, not necessarily in the correct order above "five." (if non-verbal, uses visual supports to communicate). May say "One, two, three, four, five, seven." If knows more number words than number of objects, rattles them off quickly at the end; if more objects, "recycles" number words (inflexible list exhaustion)	Verbally counts to ten (if non-verbal, uses visual supports to communicate) with some correspondence with objects, but may either continue an overly rigid correspondence or exhibit performance errors (e.g., skipping, double-counting).	Keeps one-to-one correspondence between counting words and objects (one word for each object), at least for small groups of objects laid in a line.Accurately counts objects in a line to 5 and answers the "how many" question with the last number counted, understanding that this represents the total number of objects (the cardinal principle).	Counts out objects to 5. recognises that counting is relevant to situations in which a certain number must be placed. Produces a group of 4 objects.
					Counting: Counter (10) - *Learning Trajectories*	
					Counts arrangements of objects to 10 with understanding of the cardinal principle. May be able to read and write numerals to represent 1–10. May be able to tell the number just after or just before another number, but only by counting up from 1. Verbal counting to 20 is developing (if non-verbal, uses visual supports to communicate).	
					Numeral meanings - *NCETM*	
					Children need to have the opportunity to match a number symbol with a number of things. Look for opportunities to have a range of number symbols available, e.g. wooden numerals, calculators, handwritten (include different examples of a number.	
SUBITIZING	Subitizing: Very Small Number recogniser - *Learning Trajectories*		Subitizing: Maker of Small Collections - *Learning Trajectories*	Subitizing: Small Collection Namer - *Learning Trajectories*	Subitizing: Perceptual Subitizer to 4 - *Learning Trajectories*	Subitizing: Perceptual Subitizer to 5 - *Learning Trajectories*
	Begins connecting small quantities to number words to form an explicit idea of cardinality, or "how-many-ness." Following the child's first birthday, the number words "one" and "two" are often learned. Other general terms such as "more" and "less" usually follow. Only over time do they begin to understand that all groups labelled with the same number word have the same amount.		Makes a small collection (usually 1–2 and possibly 3) with the same number as another collection (via mental model; i.e., not necessarily by matching—for that process, see Compare Number). Might also be verbal but often is not. May not recognise spatial structures at first, and may count this.	Names groups of 1, 2, and 3 with increasing accuracy. For a Maker of Small Collections (the previous level), the child may rely on matching strategies to make their small collection. In Small Collection Namer, the child is actually able to recognise small groups without relying on a model or matching strategy.	Instantly recognises collections up to 4 briefly shown and verbally names the number of items (if non-verbal, uses visual supports to communicate).	Instantly recognises briefly shown collections up to 5 and verbally names the number of items (if non-verbal, uses visual supports to communicate). Recognises and uses spatial and numeric structures beyond the situations in which they were already experienced (i.e., in which they were initially learned).
						Subitizing: Conceptual Subitizer to 5 - *Learning Trajectories*
						Verbally (if non-verbal, uses visual supports to communicate) labels all arrangements to about 5, shown only briefly, by seeing the parts and quickly knowing the whole. Conceptual subitizing refers to the ability of children to identify a whole quantity as a result of composing smaller quantities (recognised through perceptual subitizing) that make up the whole.
						Subitizing: Conceptual Subitizer to 7 - *Learning Trajectories*
						Verbally labels all arrangements to 6, then 7, when shown only briefly.

Comparison

	M4	M5	M6	M7	M8	M9
COMPARING NUMBER	More than/ Less than - *NCETM*	Comparing Number: Early Comparison - *Learning Trajectories*	Comparing Number: Perceptual Comparer - *Learning Trajectories*	Comparing Number: First-Second Ordinal Counter - *Learning Trajectories*	Comparing Number: Matching Comparer - *Learning Trajectories*	Comparing Number: Spatial Extent Estimator—Small/Big - *Learning Trajectories*
	Children need progressive experiences where they can compare collections and begin to talk about which group has more things. Initially, the groups need to be very obviously different, with one group having a widely different number of things. Collections should also offer challenges, such as including more small things and fewer large things, to draw attention to the numerosity of the comparison, i.e. the number of things, not the size of them.	Recognises that two very small collections have the “same number” by intuitively making a correspondence between the items in each collection. At this level, in certain situations, children may also put objects, words, or actions in one-to-one or many-to-one correspondence or a mixture.	Compares collections that are quite different in number (e.g., one is at least twice the other). Compares similar collections but only involving very small numbers. Compares collections using number words “one” and “two”.	Identifies the “first” and often “second” objects in a sequence.	Compares groups of 1–6 by matching.	Estimates which set is more or less if the differences are clear (e.g., one is double the other). Names a "small number" (e.g. from 1-4) for sets that cover little space and a "big number" (10-20 or more) for sets that cover a lot of space. Children classify numbers "little"/"big" idiosyncratically, and this may change with the size of the to-be estimated objects.
				Comparing Number: Early Comparer of Similar Items - *Learning Trajectories*	Comparing Number:Counting Comparer (Same Size) - *Learning Trajectories*	Comparing Number: Counting Comparer (5) - *Learning Trajectories*
				Compares collections of 1 to 4 items verbally or nonverbally (“just by looking”). The items must be the same. May compare the small collections using number words “two” and “three”, and “three” and others. Some do this even before they can accurately by using recognition of number/subitizing for these quantities. May transfer an ordering relation from one pair of collections to another.	Accurately compares via counting, but only when objects are about the same size and groups are small (up to about 5). Not always accurate when the larger collection’s objects are smaller in size than the objects in the smaller collection. Accurately counts two equal collections, but, when asked, says the collection of larger blocks has more.	Compares with counting, even when larger collection’s objects are smaller. Later, figures out how many more or less.
				Comparing Number: Early Comparer of Dissimilar Items - *Learning Trajectories*	Conservation - *NCETM*	Comparing Number: Mental Number Line to 5 - *Learning Trajectories*
				Matches small, equal collections consisting of different items, showing that they are the same number.	Children need the opportunity to recognise amounts that have been rearranged and to generalise that, if	Uses knowledge of counting number relationships to determine relative size and position when given perceptual support.
						Comparing Number: Serial Orderer to 5 - *Learning Trajectories*
						Orders quantities (dots) or numerals up to 5. Similarly orders lengths marked into units.
						Comparing Number: Ordinal Counter - *Learning Trajectories*
						Identifies and uses ordinal numbers from “first” to “tenth.”
						Comparing Number: Counting Comparer (10) - *Learning Trajectories*
						Compares with counting, even when larger collection’s objects are smaller, up to 10.
						Comparing Number: Mental Number Line to 10 - *Learning Trajectories*
						Uses internal images and knowledge of number relationships to determine relative size and position.

Composition

	M4	M5	M6	M7	M8	M9
COMPOSING NUMBER	Composing Numbers: Parts Combiner - *Learning Trajectories*		Part–whole: identifying smaller numbers within a number (conceptual subitising – seeing groups and combining to a total) - *NCETM*	Inverse operations - *NCETM*	A number can be partitioned into different pairs of numbers - *NCETM*	A number can be partitioned into more than two numbers - *NCETM*
	Recognises that sets can be combined in different orders, but may not explicitly recognise that groups are additively composed of smaller groups. The student also recognises part-whole relations in nonverbal, intuitive, perceptual situations and can nonverbally represent parts that make a whole.		Children need opportunities to see small numbers within a larger collection. 'Number talks' allow children to discuss what they see. For instance, with giant ladybirds: 'There are 5 spots altogether. I can see 4 and 1, I can see 3 and 2, and I can see 1 and 1 and 1 and 1 and 1.' Encourage exploration of all the ways that 'five' can be and look. Children are encouraged to look closely at numbers to see what else they can see. This reinforces the concept of conservation.	Children need opportunities to partition a number of things into two groups, and to recognise that those groups can be recombined to make the same total. Encourage children to say the whole number that the 'parts' make altogether.	Children need opportunities to explore a range of ways to partition a whole number. The emphasis here is on identifying the pairs of numbers that make a total. Children can do this in two ways – physically separating a group, or constructing a group from two kinds of things.	Children need opportunities to explore the different ways that numbers can be partitioned, i.e. into more than two groups. Situations to promote this include increasing the number of pots to put a given amount into, e.g. planting ten seeds into three or more pots.
			Composing Numbers: Inexact Part-Whole Recognizer - *Learning Trajectories*			Number bonds: knowing which pairs make a given number - *NCETM*
			Knows that a whole is bigger than parts, but may not accurately quantify (label with numbers). (May show intuitive knowledge of commutativity, and, later, associativity with physical groups, later in more abstract contexts, including numbers.)			Children need opportunities to say how many are hidden in a known number of things. For example: 'Five toys go into a tent, then two come out. How many are left in the tent?' The child should respond that there are still three toys in the tent.
						Composing Numbers: Composer to 4, then 5 - *Learning Trajectories*
						Knows number combinations. Quickly names parts of any whole, or the whole given the parts.
ADDING/ SUBTRACTING		Adding & Subtracting: Preverbal +/- - *Learning Trajectories*		Adding & Subtracting: Small Number +/- - *Learning Trajectories*	Adding & Subtracting: Find Result +/- - *Learning Trajectories*	Adding & Subtracting: Find Change +/- - *Learning Trajectories*
		Adds and subtracts very small collections (totals up to three), often making a collection rather than answering verbally (if non-verbal, uses visual supports to communicate).		Finds sums for Join, Result Unknown and Separate, Result Unknown problems with totals up to 5 by "counting all" with objects.	Finds sums for Join, Result Unknown problems ("You had 3 apples and get 3 more, how many do you have in all?") and part–part–whole ("There are 6 girls and 5 boys on the playground, how many children were there in all?") problems by direct modeling, counting all, with objects. Solves take-away problems by separating with objects.	Finds the missing addend (e.g., 5 + _ = 7 or 9 - _ = 3) to solve Join and Separate, Change Unknown problems by adding on or taking away objects. Compares by matching in simple situations.
					Adding & Subtracting: Make it N - *Learning Trajectories*	
					Adds on objects to "make one number into another," without needing to count from 1. Does not (necessarily) represent how many were added (this is not a requirement of this intermediate-difficulty problem type).	
MULTIPLICATION/ DIVSION	Multiplying/Dividing: Nonquantitative Sharer: Foundations - *Learning Trajectories*			Multiplying/Dividing: Beginning Grouper and Distributive Sharer - *Learning Trajectories*	Multiplying/Dividing: Grouper and Distributive Sharer - *Learning Trajectories*	Multiplying/Dividing: Concrete Modeler x/÷ - *Learning Trajectories*
	Gives some, but not necessarily an equal number to each person.			Makes small groups (fewer than 5). Shares by "dealing out," but usually only between 2 people. May not appreciate the numerical result.	Makes small equal groups (fewer than 6). Deals out equally between two or more recipients, but may not understand that equal quantities are produced.	Solves small-number multiplying problems by grouping – making each group and counting all. Solves division/sharing problems with informal strategies, using concrete objects; up to 20 objects and 2-5 people. May not understand equivalence of groups.
FRACTIONS		Fractions: Foundations: Early Proportional Thinker - *Learning Trajectories*	Fractions: Shape Equipartitioner - *Learning Trajectories*		Fractions: Half Recognizer - *Learning Trajectories*	Fractions: Unit Fraction Recognizer - *Learning Trajectories*
		Has an intuition about proportions.	Can equipartition a whole shape, such as a circle or rectangle.		Recognizes "halves" at least in continuous (e.g., area) representations, especially in the context of fair shares. Recognition of the need for ½ when sharing an odd number of objects. Intuitively and visually, combines regions that are a part of a whole, showing initial foundations for addition.	Recognizes unit fractions in simple discrete (countable) and maybe continuous (e.g., area) representations for ½, 1/3, and ¼ and understands intuitively that they are formed by dividing a whole into equal parts. Names these shares.

Pattern

	M4	M5	M6	M7	M8	M9
PATTERNING	Patterning: Foundations - *Learning Trajectories*	Continuing an AB pattern - *NCETM*	Patterning: Pattern recogniser - *Learning Trajectories*	Patterning: Patterner AB - *Learning Trajectories*	Spotting an error in an AB pattern - *NCETM*	Patterning: Patterner - *Learning Trajectories*
	Detects and uses patterning implicitly and intuitively, such as in movement activities or common nursery rhymes that repeat words and action. May be attentive to repeating patterns without recognizing them explicitly or accurately, often attending to individual attributes such as colour.	Children need the opportunity to see a pattern, to talk about what they can see, and to continue a pattern. At first, they will do this one item at a time, e.g. red cube, blue cube, red cube…verbalising the pattern helps. Children may then be asked to say what they would add next to continue it.	Recognises a simple pattern, usually ABABAB, as a pattern, even if doesn't yet name or describe it.	Recognises, describes, and builds repeating ABAB patterns. These involve the following, which many children learn in this order, although this can vary by the task.1 Fixes AB: Fills in missing element of an ABAB pattern. Duplicates AB: Duplicates ABABAB pattern (at first may have to work close to the model pattern, but eventually can build the same pattern away from the model pattern or when the model is out of sight). Extends AB: Extends AB patterns to add multiple units to the end of the pattern. This is easier for children if the pattern ends with a complete unit, but they eventually learn to	When working with AB patterns, children also need the opportunities to spot and correct errors. It is easiest to spot an extra item, then a missing item, then items swapped around. When presented with an AB pattern, children can be encouraged to describe it to make sure it is right. Then, to detect an error, they can track the pattern from the start. To begin with, children may know there is something wrong, but might not be able to say what the error is. They then might take several attempts to correct it, before being able to repair the error in one move.	Recognises, describes, and builds repeating patterns, including AB but also patterns with core units such as AAB, ABC, and AABC.
						Patterning: Pattern Translator and Unit recogniser - *Learning Trajectories*
						Translates patterns into new media or using new materials; that is, abstract and generalize the pattern. Identifies the smallest core unit of a repeating pattern. (Most research indicates this develops later, Miller et al., 2016.) In functional thinking situations (e.g., p. #), does not yet see math relationships in sets of data, level 1.

Shape and Space

	M4	M5	M6	M7	M8	M9
2D SHAPES	Shapes: Shape Matcher Identical, Sizes, & Orientations - *Learning Trajectories*		Shapes: Shape recogniser-Typical - *Learning Trajectories*	Shapes: Shape Matcher—More Shapes, Sizes, & Orientations, Combinations - *Learning Trajectories*	Shapes: Shape recogniser—Circles, Squares, and Triangles - *Learning Trajectories*	Shapes: Side recogniser - *Learning Trajectories*
	Matches familiar shapes (circle, square, typical triangle) with same size and orientation. Matches familiar shapes with different sizes. Matches familiar shapes with different orientations.		Recognises and names a typical circle, square, and, less often, triangle. May physically rotate shapes in atypical orientations to mentally match them to a prototype.	Matches a wider variety of shapes with same size and orientation. Matches a wider variety of shapes with different sizes and orientations. Matches combinations of shapes to each other.	Recognises some less typical squares and triangles and may recognise some rectangles, but usually not rhombuses (diamonds). Often doesn't differentiate sides/corners. (duplicate for Rectangles)	Identifies sides as distinct geometric objects with attributes.
					Shapes: Constructor of Shapes from Parts—Looks Like - *Learning Trajectories*	Shapes: Most Attributes Comparer - *Learning Trajectories*
					Uses manipulatives representing parts of shapes, such as sides, to make a shape that "looks like" a goal shape. May think of angles as a corner (which is "pointy").	Looks for differences in attributes, examining full shapes, but may ignore some spatial relationships.
					Shapes: Shape recogniser-All Rectangles - *Learning Trajectories*	Shapes: Corner (Vertex, Angle) recogniser - *Learning Trajectories*
					Recognises rectangles of all sizes, shapes, and orientations.	Recognises angles as separate geometric objects, at least in the limited context of "corners."
						Shapes: Shape Recognizer—More Shapes - *Learning Trajectories*
						Recognizes most familiar shapes and typical examples of other shapes, such as hexagon, rhombus (diamond), and trapezoid.
COMPOSING 2D SHAPES	Composing 2D Shapes: Separate Shapes Actor: Foundations - *Learning Trajectories*			Composing 2D Shapes: Piece Assembler - *Learning Trajectories*	Composing 2D Shapes: Picture Maker - *Learning Trajectories*	Composing 2D Shapes: Simple Decomposer - *Learning Trajectories*
	Students manipulate shapes as individuals, but usually do not combine them to compose a larger shape.			Makes pictures in which each shape represents a unique role (e.g., one shape for each body part) and shapes touch. Fills simple puzzles in which all shapes are outlined, often using trial and error.	Puts several shapes together to make one part of a picture (e.g., two shapes for one arm). Uses trial and error, and does not anticipate creation of new geometric shape. Chooses shapes using "general shape" or side length. Fills "easy" "Pattern Block Puzzles" that suggest the placement of each shape (note that in the example on the right the child is trying to put a square in the puzzle where its right angles will not fit).	Decomposes ("takes apart" into smaller shapes) simple shapes that have obvious clues as to their decomposition.
						Composing 2D Shapes: Shape Composer - *Learning Trajectories*
						Composes shapes with anticipation ("I know what will fit!"). Chooses shapes using angles as well as side lengths. Rotation and flipping are used intentionally to select and place shapes. In the "Pattern Block Puzzles" below, all angles are correct, and patterning is evident.
DISEMBEDDING SHAPES			Disembedding shapes: Intuitive Disembedder: Foundations - *Learning Trajectories*		Disembedding shapes: Simple Disembedder - *Learning Trajectories*	
			Can remember and reproduce only one or a small collection of nonoverlapping (isolated) shapes.		Identifies frame of complex figure. Finds some shapes in arrangements in which figures overlap, but not in those in which figures are embedded within others.	
3D SHAPES	3D shapes: 3D Perceiver: Foundations - *Learning Trajectories*			3D shapes: 3D Prototype recogniser - *Learning Trajectories*		
	Children can perceive 3D shapes accurately from infancy, however, this competence is limited to continuously moving objects, rather than single or even multiple static views of the same object.			Children can recognise some prototypical 3D shapes, such as the sphere and cube, using formal or informal names. However, may use 2D vocabulary to name some 3D shapes and describe solids using a variety of informal characteristics, such as "pointyness" or "slenderness."		
COMPOSING 3D SHAPES	Composing 3D shapes: Separate Blocks Actor: Foundations - *Learning Trajectories*	Composing 3D shapes: Line Maker - *Learning Trajectories*	Composing 3D shapes: Piece Assembler (3D) - *Learning Trajectories*	Composing 3D shapes: Picture Maker (3D) - *Learning Trajectories*		Composing 3D shapes: Shape Composer (3D) - *Learning Trajectories*

	Either places blocks randomly or manipulates shapes as individuals, but does not combine them to compose a larger shape. May pound, clap together, or use slide blocks or single blocks to represent an object, such as a house or truck.	Shows use of relationship of “next to” to make a (one-dimensional) line of blocks.	Builds vertical and horizontal components within a building, but within a limited range, such as building a “floor” or a simple “wall.” These, then, are two-dimensional structures.	Uses multiple spatial relations, extending in multiple directions and with multiple points of contact among components, showing flexibility in integrating parts of the structure. Produces arches, enclosures, corners, and crosses, although may use unsystematic trial and error and simple addition of pieces.		Composes shapes with anticipation, understanding what 3D shape will be produced with a composition of 2 or ore other (simple, familiar) 3D shapes. Can produce arches (with vertical interior space), enclosures (with internal horizontal space), corners, and crosses systematically. Builds enclosures and arches several blocks high. Later in this level, children add depth to make 3D structures, and they add roofs across structures multiple blocks high (but they may have no internal spaces).
	Composing 3D shapes: Stacker - *Learning Trajectories*	Composing 3D shapes: Same Shape Stacker - *Learning Trajectories*				
	Shows use of the spatial relationship of “on” to stack blocks, although choice of blocks may be unsystematic.	Shows use of relationship of “on” to stack congruent blocks, or those that show a similarly helpful relationship to make stacks or lines.				
SPATIAL VISUALISATION		Spatial Visualization: Concrete Slider, Flipper, Turner - *Learning Trajectories*		Spatial Visualization: Simple Slider and Turner - *Learning Trajectories*		Spatial Visualization: Beginning Slider, Flipper, Turner - *Learning Trajectories*
		Can move shapes to a location by physical trial and error.		Slides and turns objects accurately in easy tasks, guided by an early intuition that starts the motion and then adjusts (the motion, direction, or amount) in real time as the motion is carried out.		Uses the correct motions guided by more developed intuition, but not always accurate in direction and amount (adjusts these with trial and error). Knows a shape has to be flipped to match another shape, but flips it in the wrong direction.
SPATIAL ORIENTATION	Spatial Orientation: Path Integrater - *Learning Trajectories*	Spatial Orientation: Place Learner - *Learning Trajectories*	Spatial Orientation: Local-Self Framework User - *Learning Trajectories*		Spatial Orientation: Small Local Framework User - *Learning Trajectories*	Spatial Orientation: Local Framework User - *Learning Trajectories*
	Remembers and can repeat movements they have made including the approximate distances and directions.	Creates “mental maps” by storing locations, distances, and directions to landmarks and solves spatial problems. Uses the walls of a room as a frame of reference; uses spatial vocabulary, such as "in," "on," and "under," along with vertical directionality terms as "up" and "down."	Uses distant landmarks to find objects or location near them, even after they have moved themselves relative to the landmarks, if the target object is specified ahead of time. Orients a horizontal or vertical line in space (Rosser, Horan, Mattson, & Mazzeo, 1984). Uses spatial vocabulary to direct attention to spatial relations, including more difficult terms such as "beside" and "between."		Locates objects after movement, even if target is not specified ahead of time. Searches a small area comprehensively, often using a circular search pattern. Uses words referring to frames of reference such as "in front of" and "behind" or "left" and "right." In meaningful graphing contexts, extrapolates lines from positions on both axes (like a coordinate grid) and determines where they intersect.	Locates objects after moving, maintaining the overall shape of the arrangement of objects. Represents objects' positions relative to landmarks (e.g., about halfway in between two landmarks) and keeps track of own location in open areas or mazes. Uses spatial vocabulary to direct attention to spatial relations. Uses coordinate labels in simple situations such as games.

Measures

	M4	M5	M6	M7	M8	M9
LENGTH	Length: Length Senser: Foundations - *Learning Trajectories*		Length: Length Quantity recogniser - *Learning Trajectories*		Length: Length Direct Comparer - *Learning Trajectories*	Length: Length Indirect Comparer - *Learning Trajectories*
	Makes simple comparisons of length intuitively (similar to what we saw in Subitizing).		Identifies length/distance as an attribute. May understand length as an absolute descriptor (e.g., all adults are tall), but not as a comparative (e.g., one person is taller than another). May compare noncorresponding parts of shape in determining side length.		Physically aligns two objects to determine which is longer or if they are the same length. Uses terms: long, longer, longest.	Compares the length of two objects by representing them with a third object. Uses terms: long, longer, longest, short, shorter, shortest. When asked to measure, may assign a length by guessing or moving along a length while counting (without equal-length units). May be able to measure with a ruler, but often lacks understanding or skill (e.g., ignores starting point).
						Length: Serial Orderer to 5 - *Learning Trajectories*
						Orders lengths, marked in 1 to 5 units. Also, can compare unmarked lengths that are clearly different using broad categories ("big" and "small") and so can order 3 to 5 such objects but only by trial-and-error. With an increase in working memory, begins to build a mental image of the final ordering in which the lengths increase "bit by bit" with each successive length the smallest increase. This leads to more accurate and somewhat more efficient ordering. (This level develops in parallel with "End-to-End Length Measurer".)
AREA		Area: Area Senser: Foundations - *Learning Trajectories*		Area: Area Quantity recogniser - *Learning Trajectories*	Area: Physical Coverer and Counter - *Learning Trajectories*	Area: Complete Coverer and Counter - *Learning Trajectories*
		Even children in the early stages are sensitive to area. However, may not explicitly recognise area as an attribute (separate from general size, such as "small" and "big") for some time. If asked to fill in a rectangle, preschoolers may just draw approximations of circles (Mulligan, Prescott, Mitchelmore, & Outhred, 2005). Uses side matching strategies in comparing areas.		Perceives the amount of two-dimensional space and can make intuitive comparisons. However, when asked to compare, may compare lengths more than areas because lengths are salient and familiar to them (e.g., compare one side of one piece of paper to the side of another) or make estimates based on a "length plus (not times) width" intuition. However, may compare areas correctly if the task suggestions superposition (putting one on top of the other). Asked to partition a space into squares or copy an image of a rectangle partitioned into an array (rows and columns), may simply draw squares (usually!) inside the rectangle or other types of shapes or short paths on or around the rectangle.	Prompted to measure, attempts to cover a rectangular space with physical tiles. However, doesn't organize or structure the 2D space without considerable perceptual support, such as a grid that outlines each individual unit. In drawing (or imagining and pointing to count squares as units of area), represents only certain aspects of that structure, such as approximately rectangular shapes next to one another. Makes comparison areas based on simple, direct comparisons (e.g., a child places one sheet of paper over another piece of paper to select the sheet that covers more space).	Draws a complete covering of a specific region without gaps or overlaps and in approximations of rows. When provided with more than the total number of physical tiles needed, can build a region of specified area (e.g., build a rectangle with an area of 12 from a pile of 20 tiles).
VOLUME	Volume: Volume Senser: Foundations - *Learning Trajectories*	Volume: Volume Quantity recogniser - *Learning Trajectories*		Volume: Volume Filler - *Learning Trajectories*		
	Sensitive to volume even in the early stages; however, they may not for some time explicitly recognize volume as an attribute (separate from general size, such as "small" and "big").	Identifies capacity or volume as attribute. Builds with blocks, associating more blocks with terms like "big" and fewer blocks with terms like "small."		Can compare two containers by pouring one into the other (although can be confused at "which holds more" at first). Fills a container using another (smaller container) and counts the number needed to completely fill the larger container (but may not use accurately filled scoops and may not focus on quantifying the total volume or capacity). In packing situations, places cubes into a rectangular box to fill it. Eventually packs entire box with cubes in an organized way. Compares objects by physically or mentally aligning; refers to at least two dimensions of objects. May be able to compare two containers using a third container and transitive reasoning.		
ANGLE AND TURN	Angle and Turn measurement: Angle and Turn Senser: Foundations - *Learning Trajectories*		Angle and Turn measurement: Intuitive Angle Builder - *Learning Trajectories*			Angle and Turn measurement: Implicit Angle User - *Learning Trajectories*
	Students are sensitive to angles-as-turning, both turning objects and their own body. See more at the first three levels of the Spatial Orientation and the first level of Spatial Visualization.		Intuitively uses some angle measure notions in everyday settings, such as building with blocks, solving puzzles, and walking.			Uses angles and, at least implicitly, some angle measure concepts, such as parallelism and perpendicularity—in physical alignment tasks, construction with blocks, or other everyday contexts. May identify corresponding angles of a pair of congruent triangles using physical models. Uses the word "angle" or other descriptive vocabulary to describe some of these situations.
CLASSIFICATION AND DATA ANALYSIS		Classification & Data: Similar/Dissimilar Maker - *Learning Trajectories*		Classification & Data: Simple Sorter - *Learning Trajectories*	Classification & Data: Sorter by Similar Attributes - *Learning Trajectories*	Classification & Data: Consistent, Flexible Sorter - *Learning Trajectories*
		Forms sets in which objects in each set are identical and objects in the other sets are different. Intuitively forms groups with objects that are similar on some attributes (may be mixed and inconsistent), but not necessarily identical.		Follows verbal rules for sorting scaffolded by an adult. (These may be made with shifting criteria; nevertheless, they play an essential role in number, through the unitizing process.) Can "fix" a simple sort with mistakes.	Sorts objects according to an explicit attribute (although still may decide to switch attributes during the sorting). The end result may appear to reflect adult categorizations, but often has a different basis, such as general resemblance.	Sorts consistently by a single attribute and re-classifies by different attributes. Sorts consistently and exhaustively by an attribute, given or created, and uses the terms "some" and "all."
						Classification & Data: Data Case Viewer - *Learning Trajectories*
						Associates a value with an individual case. Uses numeric data to identify largest/smallest cases. Before this level, children may be "pointers" in which data records point to the entire event ("We talked about favorite colors"). They use it like string tied around a finger, to remember that they did something.